

# Correlations, Decoherence, Dissipation, and Noise in Quantum Field Theory \*

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## Abstract

The statistical mechanical properties of interacting quantum fields in terms of the dynamics of the correlation functions are investigated. We show how the Dyson - Schwinger equations may be derived from a formal action functional, the  $n$ -particle irreducible ( $nPI, n \rightarrow \infty$ ) or the ‘master’ effective action. It is related to the decoherence functional between histories defined in terms of correlations. Upon truncation of the Dyson - Schwinger hierarchy at a certain order, the master effective action becomes complex, its imaginary part arising from the higher order correlation functions, the fluctuations of which we define as the correlation noises of that order. Decoherence of correlation histories via these noises gives rise to classical stochastic histories. Ordinary quantum field theory corresponds to taking the lowest order functions, usually the mean field and the 2-point functions. As such, our reasoning shows that it is an effective theory which can be intrinsically dissipative. The relation of loop expansion and correlation order as well as the introduction of an arrow of time from the choice of boundary conditions are expounded with regard to the origin of dissipation in quantum fields. Relation with critical phenomena, quantum transport, molecular hydrodynamics and potential applications to quantum gravity, early universe processes and black hole physics are mentioned.

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# 1 Introduction and Summary

## 1.1 Background

The present work is a continuation of our systematic investigation since 1986 on the statistical and stochastic natures of quantum field theory. It has developed in three stages incorporating three key ideas. Our 1988 work [1] on non-equilibrium quantum fields studied the kinetic theory aspects of interacting quantum field theory. The closed time path (CTP) [2]  $n$ -particle irreducible (nPI) [3] effective action for the correlation functions was applied to the example of a  $\lambda\phi^4$  theory to derive the quantum field equivalent of the BBGKY hierarchy of equations. We derived the relativistic quantum field Boltzmann equation and explained the origin of dissipation in a three-loop perturbation analysis. In there, the first key concept of *correlation and dissipation* in the Boltzmann context was expounded.

Investigating the missing link with noise and fluctuations which are expected to accompany dissipation [4] led to the second stage of development which involved the application of quantum open system concepts [5] and influence functional (IF) techniques [6] to quantum Brownian models (QBM) [7] and quantum field theory [8, 9]. There it was found that the effect of an environment on the system can be best depicted by an influence action which contains a dissipative kernel and a noise kernel as its real and imaginary parts, the two always balanced by a fluctuation-dissipation relation [10]. The noise is generally colored, being determined by the spectral density of the environment and the type of coupling between the system and the environment. The influence functional formalism makes it possible to derive from first principles the nature of quantum noise in terms of the fluctuations of quantum fields [11, 12] and a Langevin or Fokker-Planck equation for the dissipative dynamics of the system driven by such stochastic sources. In there, the second key concept of *noise and fluctuations* and their relation with dissipation are explored in the Langevin context.

The third key concept is *decoherence*, which predicates the transition of a system from quantum to classical. In the environment-induced decoherence scheme [13] its effect can be seen as a direct consequence of the system's coupling to an environment and the coarse-graining of the environment. The formulation in terms of consistent histories [14] is an important conceptual advance in the interpretation of quantum mechanics. These programs in decoherence relate decoherence in the system to noise and fluctuations in the environment. Applying the consistent history scheme to correlations of histories, we formulated a new way to describe decoherence and quantum to classical transition for a closed system, where the coarse-graining is determined by the correlation order. It is, in our opinion, more natural than the open system decoherence schemes where the system- environment separation and the coarse-graining (by averaging out some variables in the environment or by the use of smearing or gate functions in path-integrals) are put in by hand. Closing this line of development of ideas, we further showed [11] that the closed time path formulation of field theory is largely equivalent to the influence functional formulation from which one can easily derive the Langevin equation driven by a stochastic source (noise) related to the fluctuations of the quantum fields for the dissipative dynamics of an open system. Our present work is

a further development of our previous work [1, 15, 11], linking up the three pairs of key relations amongst *correlation*, *decoherence*, *dissipation* and *noise*. The interconnection of these effects has been emphasized before by Hu [16] and Gell-Mann and Hartle [17].

As background, some key ideas described above have been explored in the context of quantum mechanics [18], gravitation and cosmology [19, 20, 21, 22]. On a grander scale, our program here strives to establish a statistical mechanics of quantum fields. As we shall see, the physical meaning of many common techniques and approximation schemes in perturbative field theory can be truly understood only by bringing in some basic concepts and techniques of statistical mechanics [23, 24, 25], kinetic theory [26, 27, 28], stochastic mechanics [29, 30], critical dynamics [31, 32], and hydrodynamics [33, 34]. For example, the construction of the effective action [35], which describes the dynamics of the mean field, falls under the general pattern of identification of a subdynamics [?] through coarse graining and truncation of a more complete underlying theory. The meaningfulness of such a division depends on the identifiability and viability of different space, time and interaction scales of the characteristic physical processes involved [37]. Of course, this is just one instance of the kind of process by which many body theory and kinetic theory are reduced to hydrodynamics [34], or, for that matter, any microscopic description of a system is coarse-grained into some macroscopic dynamics of its collective modes [38].

## 1.2 Summary

We shall now summarize the main themes and key findings of this paper.

### 1.2.1 Main Themes

In this paper we start from the thesis that the full dynamics of an interacting quantum field may be described by means of the Dyson- Schwinger equations governing the infinite hierarchy of Wightman functions which measure the correlations of the field. We show how this hierarchy of equations can be obtained from the variation of the infinite particle irreducible, or ‘master’ effective action. Truncation of this hierarchy gives rise to a quantum subdynamics governing a finite number of the correlation functions (which constitute the ‘system’) and expression of the higher order correlation functions (which constitute the ‘environment’) in terms of the lower-order ones by functional relations (‘slaving’ or ‘factorization’) induces dissipation in the dynamics of the subsystem driven by the stochastic fluctuations of the environment, which we call the ‘correlation noises’. These two aspects are related by the fluctuation-dissipation relation. This is the quantum field equivalent of the BBGKY hierarchy in Boltzmann’s theory. Any subsystem involving a finite number of correlation functions defines an effective theory, which is, by this reasoning, intrinsically dissipative. The relation of loop expansion and correlation order is expounded. We see that ordinary quantum field theory which involves only the mean field and a two-point function, or any finite-loop effective action in a perturbative theory are, by nature, effective theories which possess these properties. Histories defined by lower-order correlation functions can be decohered by the noises from the higher order functions and acquire classical stochastic

attributes. The present scheme invoking the correlation order is a natural way to describe the quantum to classical transition for a closed system as it avoids *ad hoc* stipulation of the system-environment split. It is through decoherence that the subsystem variables become classical and the subdynamics becomes stochastic.

### 1.2.2 Key Points

#### *Fluctuations in Mean Field and Correlations; Lessons from Critical Dynamics and Kinetic Theory*

One useful departure point from the conventional treatment is to view the so-called ‘mean’ field not as the actual expectation value of the field, but rather as representing the local value of the field within one particular history. Quantum evolution encompasses the coherent superposition of all possible histories [14]. These quantities are subject to fluctuations. The theory may be enlarged by including some correlation functions as independent variables along with the ‘mean’ field. These correlation functions will be subject to fluctuations of their own. We can recognize the inadequacy of theories built on mean field and its fluctuations from examining some familiar physics problems.

One area where the correlations of fields play a central role is critical dynamics. There, one focuses on the evolution of some ‘order parameter’ whose mean value obeys a Landau - Ginzburg equation [23]. In practice, however, the actual value of the order parameter can have strong fluctuations around the mean value as the critical point is approached, and the Landau - Ginzburg equation must be generalized to include stochastic terms such as is used in the Cahn - Hilliard theory [39]. The difference in our treatment is that the stochastic source (noise) term is not put in by hand, but is derived from the given structures of the field theory in question. Equivalently, it is possible to describe the evolution in terms of a Fokker - Plank equation [40], which may be used to derive a hierarchy of equations for the correlation functions. For practical applications this hierarchy must be truncated, as was done in Langer’s treatment [41]. One of us has used the CTP nPI method in quantum field theory to the problem of spinodal decomposition before [42]. Although the effect of fluctuations on the correlation functions has not been explored in full (see, however, [43, 44]), this step is nevertheless common in dilute gas dynamics [45].

In the dynamics of a dilute gas [26, 27] the exact Newton’s or Hamilton’s equations for the evolution of a many body system may be translated into a Liouville equation for the distribution function or the BBGKY hierarchy for the sequence of partial (n- particle) distributions. This reformulation is only formal, which involves no loss of information or predictability. A more realistic description of the dynamics corresponding to physical conditions comes from a truncated BBGKY hierarchy, where the higher order distributions are substituted by functionals of the lower order, and ultimately, the one- particle distributions. Constructed perturbatively, this effective theory follows only approximately the actual dynamics. Moreover, these functionals embody some relevant boundary conditions (such as the ‘weakening of correlations’ hypothesis [26]), which make them noninvariant upon time rever-

sal. This is how dissipation in the explicitly irreversible Boltzmann's equation appears. On closer examination, it is seen that the one- particle distribution function itself describes only the mean number of particles within a certain location in phase space; the actual number is also subject to fluctuations. From the average size of the equilibrium fluctuations, which can be determined from Einstein's formula, and the dissipative element of the dynamics, which is contained in the collision integral, it is possible to compute the stochastic driving force consistent with the fluctuation- dissipation relation near equilibrium [45].

### *Dyson-Schwinger Hierarchy; the Master Effective Action, Truncation and Slaving*

We want to describe a quantum field in terms of the mean field and the (infinite number of) correlation functions. Our starting point shall be the well-known fact that the set of all Wightman functions (time ordered products of field operators) determines completely the quantum state of a field [53]. Instead of following the evolution of the field in any of the conventional representations (Schrödinger, Heisenberg or Dirac's), we shall focus on the dynamics of the full hierarchy of Wightman functions. To this end it is convenient to adopt Schwinger's "closed time-path" techniques [2], and consider time ordered Green functions as a subset of all Green functions path- ordered along a closed time loop (see below). The dynamics of this larger set is described by the Dyson - Schwinger equations [50].

We will first show that the Dyson- Schwinger hierarchy may be obtained via the variational principle from a functional which we call the 'Master Effective Action' (MEA). This is a formal action functional where each Wightman function enters as an independent variable. We will then show that any field theory based on a finite number of (mean field plus ) correlation functions can be viewed as a subdynamics of the Dyson- Schwinger hierarchy. The specification of a subdynamics involves two steps: first, a finite set of variables from the original hierarchy is identified to be the 'relevant' [?] variables (which constitute the subsystem). Second, the remaining 'irrelevant' (or 'environment') variables are *slaved* to the former. Slaving (or 'factorization' in the Boltzmann theory) means that irrelevant variables are substituted by set functionals of the relevant variables. The process of extraction of a subdynamics from the Dyson- Schwinger hierarchy has a correlate at the level of the effective action, where the MEA is truncated to a functional of a finite number of variables. The finite effective actions so obtained (influence action [6]) are generally nonlocal and complex, which is what gives rise to the noise and dissipation in the subdynamics. Moreover, since the slaving process generally involves the choice of an arrow of time, it leads to irreversibility in the cloak of dissipation in the subdynamics [?].

### *Decoherence and Noise, Fluctuations and Dissipation*

Under realistic conditions, one may not be as much concerned with the full quantum evolution of the field as with the development of 'classical' theories where fields are described as c-numbers, plus perhaps a small number of correlation functions to keep track of fluctuations. These classical theories represent the physically observable dynamics after the process

of decoherence [13, 14] has destroyed or diminished the coherence of the field. In any case, no actual observation could disclose the infinite number of degrees of freedom of the quantum field, and therefore any conceivable observational situation may be described in the language of a suitable complex ‘classical’ theory in this sense.

Decoherence is brought by the effect of a coarse-grained environment (or ‘irrelevant’ sector) on the system (or the ‘relevant’ sector). In simple models, this split is imposed by hand, as when some of the fields, or the field values within a certain region of spacetime, are chosen as relevant. Here, we will follow the approach of our earlier work on the decoherence of correlation histories [15]. There is no need to select *a priori* a relevant sector within the theory. Instead, we shall seek a natural criterium for successive truncations in the hierarchy of correlations, the degree of truncation depending on the stipulated accuracy of measurement which can be carried out on the system. In this framework, decoherence occurs as a consequence of the fluctuations in the higher order correlations and results in a classical dissipative dynamics of the lower order correlations.

Here, we adopt the consistent histories formulation of quantum mechanics [14] for the study of the quantum to classical transition problem. We consider the full evolution of the field described by the Dyson- Schwinger hierarchy as a fine- grained history while histories where only a finite number of Wightman functions are freely specified (with all others slaved to them) are therefore coarse- grained. We have shown that the finite effective actions obtained for the subsystems of lower-order correlations are related to the decoherence functional between two such histories of correlations [15], its acquiring an imaginary part signifies the existence of noise which facilitates decoherence. Thus decoherence of correlation histories is a necessary condition for the relevance of the c-number theory as a description of observable phenomena.

It can be seen that if the c-number theory which emerges from the quantum subdynamics is dissipative, then it must also be stochastic.<sup>1</sup> From our correlation history viewpoint, the stochasticity is in fact not confined to the field distributions– the correlation functions would become stochastic as well [45].

Following Feynman and others [6] in their illustration of how noise can be defined from the influence functional, we can relate the imaginary part of the finite effective actions describing the truncated correlations to the auto-correlation of the stochastic sources, i.e., correlation noises, which drive the c-number fields and their correlation functions via the Langevin- type equations. From the properties of the complete (unitary) field theory which constitutes the closed (untruncated) system, one can show that the imaginary part of the effective action is related to the nonlocal part of the real part of the effective action which depicts dissipation. This is of course where the fluctuation- dissipation theorem for non-equilibrium systems

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<sup>1</sup> Because the fundamental variables are quantum in nature, and therefore subject to fluctuations, a classical, dissipative dynamics would demand the accompaniment of stochastic sources (in agreement with the ‘fluctuation - dissipation theorems’, for, otherwise, the theory would permit unphysical phenomena as the damping away of zero - point fluctuations.) Of course, these uncontrollable fluctuations may be seen at the origin of many phenomena where structure seems to spring ‘out of nothing’, such as the nucleation of inhomogeneous true vacuum bubbles in a supercooled false vacuum, or the development of inhomogeneities out of a homogeneous early universe.

originates [12].

We thus see once again the intimate connection amongst the three aspects of the theory, decoherence, dissipation, and fluctuations [16, 17], now manifesting in the hierarchy of correlations which defines the subsystems.

This paper is organized as follows. The next subsection contains a brief statement of the fluctuation- dissipation theorem in two variant forms, described here to clarify our meaning of this relation. Section 2 is a quick overview of functional methods in causal quantum field theory based on [46, 1] (See references therein for sources and details.)

In Section 3 we present a formal construction of the Master Effective Action. We compute it with the background field method, and discuss its relation with the finite theories by truncation and slaving. In Section 4 we introduce decoherence in histories and discuss the relationship of the master effective action and its truncations to the decoherence functional. As illustration, we give a brief description of the decoherence of the mean field in a self interacting field theory. This example is along the line of development in [11] and [48].

Section 5 is devoted to the new concept of ‘correlation noise’, which measures the fluctuations of the propagators themselves due to the truncation of higher correlations. We present a simple example where ‘correlation noise’ can be derived through direct physical arguments, and then show how these results follow rigorously as a simple case of the formal theory developed in sections 3 and 4.

We conclude the paper with some discussion of possible applications of this formalism and viewpoint in effective field theory, and quantum statistical processes in the early universe and black holes in Section 6.

### 1.3 The Fluctuation - Dissipation Relation (FDR)

Given the relevance of the FDR to the discussion below, it is useful to spend a few moments clarifying our meaning. Let us consider a near equilibrium system described by a single variable  $x$ ; let the entropy of the system be  $S(x)$ , and call  $y = -dS/dx$  the corresponding ‘thermodynamic force’. Let  $x$  vanish in equilibrium. A macroscopic fluctuation can be absorbed into a redefinition of the equilibrium; as a first order approximation, we may write  $\dot{x} = -ay$ . The condition that entropy should be non-decreasing in the process implies that  $a$  should be positive. A microscopic fluctuation which induces deviations from the average constitutes a stochastic force  $j$ , yielding a stochastic equation  $\dot{x} = -ay + j$ . One can close the system of evolution equations by assuming the linear law  $y = bx$ . Again, because  $S$  is maximum at the origin,  $b \geq 0$ . We thus have a Langevin equation  $\dot{x} = -cx + j$ , where  $c = ab$  is positive. This leads immediately to

$$x(t) = x(0)e^{-ct} + \int_0^t dt' e^{-c(t-t')} j(t') \quad (1.1)$$

In the literature there are two different results under the heading of ‘Fluctuation - Dissipation Theorem’, obtained by manipulating Eq. (1.1) in two different ways. The ‘Landau

- Lifschitz' result [23] is obtained by multiplying this equation by  $x(0)$ , taking the average over the stochastic forces  $j$  (assuming the average  $\langle x(0)j(t) \rangle = 0$ ), and integrating over time, leading to the Green - Kubo formula

$$c^{-1} = (\langle x^2(0) \rangle)^{-1} \int_0^\infty dt \langle x(0)x(t) \rangle \quad (1.2)$$

In this work, we shall be more concerned with the second customary formulation of the FDR, which goes back to Callen and Welton's original work [10]. Squaring both sides of Eq. (1.1), and taking the ensemble average, assuming as before that  $\langle x(0)j(t) \rangle = 0$ ,  $\langle j(t)j(t') \rangle = \sigma^2 \delta(t - t')$  and that  $\langle x^2(t) \rangle$  remains independent of  $t$  at its equilibrium value, we find

$$\sigma^2 = 2c \langle x^2 \rangle \quad (1.3)$$

Thus, the FDR determines the statistics of the random source  $j$  from the given dissipation coefficient  $c$  and the equilibrium fluctuations.

The actual result quoted by Callen and Welton reads as follows. Define at each frequency the impedance  $Z$  from  $j(\omega) = i\omega Z x(\omega)$ , and the resistance  $R = \text{Re}Z$ . In general the noise  $j(t)$  will be colored, but time translation invariant, in the sense that  $\langle j(\omega)j(\omega') \rangle \sim \delta(\omega + \omega')$ . Let us call  $\sigma^2 \equiv \langle j(t)^2 \rangle$ . Then Callen and Welton's FDR states that

$$\sigma^2 = \frac{2}{\pi} \int_0^\infty d\omega R(\omega) f(\omega, T) \quad (1.4)$$

where  $f$  is an universal function depending only on temperature

$$f(\omega, T) = \hbar\omega \left( \frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} \right) \quad (1.5)$$

In quantum field theory this formula is equivalent to the KMS condition [49]. Indeed, from the definition of the impedance we have

$$\sigma^2 = \frac{1}{(2\pi)} \int_0^\infty d\omega \omega^2 |Z(\omega)|^2 G_1(\omega); \quad (1.6)$$

the KMS condition further links the Hadamard and the Jordan propagators, namely

$$G_1 = \frac{2}{\omega} f(\omega, T) G(\omega) \text{sgn}(\omega) \quad (1.7)$$

Finally, from  $G_{ret} = -iG\theta(t - t')$  and  $G_{ret} = (i\omega Z)^{-1}$  we conclude

$$G(\omega) = \frac{2\text{sgn}(\omega)}{\omega} \frac{R}{|Z(\omega)|^2}, \quad (1.8)$$

just as dictated by the Callen and Welton expression.



## 2 Functional Methods and the Dyson - Schwinger Hierarchy

Our goal in this section is to show how a formal Effective Action may be defined, which generates the whole hierarchy of Dyson - Schwinger's equations[50]. We shall use Schwinger's CTP techniques [2], extending earlier works on higher-particle-irreducible effective actions [3]. We shall begin by recapitulating the train of thought which led us to this particular formulation of quantum field theory.

The effective action usually appears in quantum field theory textbooks in the context of perturbative expansions [51]. As it is by now a common procedure, an efficient way to generate the Feynman Green functions of a quantum field is from the variational derivatives of a generating functional. The generating functional itself admits the path integral representation

$$e^{iW[J]} = \int D\Phi e^{i\{S[\Phi] + \int d^4x J(x)\Phi(x)\}} \quad (2.1)$$

The coefficients in the Taylor expansion of  $W$  with respect to  $J$  are the connected Feynman graphs of the (scalar) field theory  $\Phi$ . In particular, we may introduce the 'background' or 'mean' field

$$\phi(x) = \frac{\delta W[J]}{\delta J} \quad (2.2)$$

A more efficient representation of the Green functions is afforded by the Legendre transformation of  $W$

$$\Gamma[\phi] = W[J] - \int d^4x J(x)\phi(x) \quad (2.3)$$

since the Taylor expansion of  $\Gamma$  involves only one particle irreducible (1PI) Feynman graphs. Actually, the physical meaning of the 'effective action'  $\Gamma$  goes beyond this: for a constant background,

$$\Gamma[\phi] = - \int d^4x V[\phi], \quad (2.4)$$

and the 'effective potential'  $V$  is the actual free energy density of the quantum field. Thus it is tempting to consider the inverse of (2.2) as the equation of motion for the background field

$$\frac{\delta \Gamma}{\delta \phi(x)} = -J(x) \quad (2.5)$$

However, this equation is generally complex. This property follows from the fact that the 'background' field is not a true expectation value, but rather a matrix element of the field between IN and OUT states [52].

We can get around this difficulty by adopting Schwinger's 'closed time-path' formalism. Let us couple the field to an external source  $J(x)$ , and adopt the interaction picture where states evolve according to the evolution operator

$$U(t) = T(e^{i \int_{-\infty}^t J(x)\Phi(x)}) \quad (2.6)$$

where  $T$  stands for time ordering. Then, if the state in the distant past is  $|IN\rangle$  (not necessarily a vacuum state), the expectation value of the field at time  $t$  is

$$\phi(x) = \langle IN | \tilde{T}(e^{-i \int^t d^4x' J(x')\Phi(x')}) \Phi(x) T(e^{i \int d^4x'' J(x'')\Phi(x'')}) | IN \rangle \quad (2.7)$$

where  $\tilde{T}$  stands for anti-temporal ordering. This may be rewritten as

$$\phi(x) = \frac{\delta}{\delta J(x)} e^{iW[J,J']} |_{J' \equiv J} \quad (2.8)$$

where  $W$  is the CTP generating functional

$$e^{iW[J,J']} = \langle IN | \tilde{T}(e^{-i \int d^4x' J'(x')\Phi(x')}) T(e^{i \int d^4x'' J(x'')\Phi(x'')}) | IN \rangle \quad (2.9)$$

The time integrals now extend to the far future. The generating functional admits a path integral representation,

$$e^{iW[J,J']} == \int D\Phi D\Phi' e^{i\{S[\Phi] - S[\Phi'] + \int d^4x (J(x)\Phi(x) - J'(x)\Phi'(x))\}} \quad (2.10)$$

Thus, we have been led to formally doubling the degrees of freedom and integrating over pairs of histories  $(\Phi, \Phi')$  which converge in the far future. In condensed notation

$$e^{iW[J_a]} = \int D\Phi^a e^{i(S[\Phi^a] + J\Phi)} \quad (2.11)$$

where the index  $a$  indicates a doubling into unprimed and primed quantities (with an opposite sign for the latter).  $S[\Phi^a] = S[\Phi] - S[\Phi']^*$  (complex conjugation applies if an  $i\epsilon$  term has been included to enforce the boundary conditions), and the 'internal' index  $a$  is lowered with the 'metric'  $g_{ab} = \text{diag}(1, -1)$ . We have also omitted summation signs over continuous indexes, thus

$$J\Phi \equiv \int d^4x J_a(x)\Phi^a(x) \quad (2.12)$$

We can actually define two background fields by taking variations with respect to either source; the physical situation corresponds to the case when both sources agree. The closed time-path effective action (CTPEA) is the double Legendre transform of the generating functional

$$\Gamma[\phi^a] = W[J_a] - \int d^4x J_a(x)\phi^a(x) \quad (2.13)$$

The equations of motion now form a coupled system

$$\frac{\delta\Gamma}{\delta\phi^a(x)} = -J_a(x) \quad (2.14)$$

They are identical to each other when  $J = J'$  is the physical external source, and they admit a solution where  $\phi = \phi'$  is the physical mean field. Of course, both background fields must be identified only after the variational derivative has been computed [46]. As opposed to the IN-OUT formulation, the equations of motion (2.14) are always real and causal whereas in cases where the IN-OUT effective action becomes complex, the IN-IN effective action leads to dissipative evolution. This fact underlines that the doubling of degrees of freedom was, after all, essential, since no action functional of the physical mean field alone could possibly lead to the right (dissipative) equations of motion.

Breaking of time symmetry in the mean field equations of motion also points to the fact that the mean field cannot be considered as a closed system. Indeed, as we get to understand the problem better, and as we shall show in this and related work, ordinary quantum field theory based on mean field and fluctuations is an open system. This is because the mean field interacts, in any non-linear field theory, with its own quantum (and thermal) fluctuations, which act as an ‘environment’ [47]. In principle it is possible to shift the division between the system and its environment by including into the former some of the correlations describing those fluctuations. This is analogous to Langer’s approach to critical dynamics [41], and has been carried out in field theory by many authors, namely Dahmen and Jona - Lasinio, Cornwall, Jackiw and Tomboulis, Norton [3], and, in the context of the CTP approach, the present authors [1] [42]. Not surprisingly, this step leads to a further compression of the perturbative series; by including two point functions as independent variables, we obtain a generating functional whose Taylor expansion contains only two-particle irreducible (2PI) Feynman graphs. Generalization of this formalism shall be described in detail in the next section.

However, there is a physical issue which is not resolved by extending to higher irreducible effective actions, namely, how to describe the dynamics of the field from the vantage point of a local observer. A possible answer is suggested by physical reasoning inspired by Onsager and Landau, namely, we obtain the relevant dynamics by including stochastic terms in the mean field equations of motion. These stochastic sources generate dissipative dynamics in the mean field equations which are related to the known quantum and thermal fluctuations of the field through the fluctuation- dissipation relations. The main thrust of this paper is precisely to show how the CTP effective action formalism may be used to explicate these physical ideas and to construct a consistent theory valid beyond the linear response regime (no equilibrium initial conditions need be assumed).

Recognition of the stochastic nature of semiclassical evolution also helps to dispel the appearance of redundancy in the CTPEA. Let us observe that quite generally the CTPEA obeys  $\Gamma[\phi, \phi] = 0$  and  $\Gamma[\phi, \phi'] = -\Gamma[\phi', \phi]^*$ . Therefore, if we separate the CTPEA in its real and imaginary parts, we find that the former is an odd function of  $\phi$  and  $\phi'$ , while the latter is even. If we consider only linearized fluctuations  $\Delta\phi^a$  around an extremum of the CTPEA, i. e., a solution of the mean field equations of motion, the CTPEA must take the form

$$\Gamma[\Delta\phi^a] = \frac{1}{2} \int d^d x d^d x' \{ -[\Delta\phi](x) \mathcal{D}(x, x') \{ \Delta\phi \}(x') + i[\Delta\phi](x) N(x, x') [\Delta\phi](x') \} \quad (2.15)$$

where  $[\Delta\phi] = (\Delta\phi - \Delta\phi')$ ,  $\{ \Delta\phi \} = (\Delta\phi + \Delta\phi')$ , and  $\mathcal{D}$  and  $N$  are two non-local kernels, to be further discussed below. The seeming redundancy appears because  $N$  does not contribute to the mean field equations of motion, which take the simple form

$$\int d^d x' \mathcal{D}(x, x') \Delta\phi(x') = 0 \quad (2.16)$$

after identifying  $\Delta\phi' = \Delta\phi$ . The answer lies, of course, in that  $N$  contains the information on the *departure* of the actual evolution from mean field behavior; this deviation may be accounted for by coupling the fluctuations to a stochastic external force [6]. We shall return to this point after introducing the Decoherence Functional below.

Bringing together all the different strands, we arrive at the conclusion that a suitable formulation of quantum field theoretic evolution should incorporate both the CTP technique, to allow for causal evolution, and the higher particle irreducible techniques, to obtain a self consistent dynamics of correlations. We will see that any quantum field theory based on a finite subset of correlation functions (which defines an open system – the truncation rendering it an effective theory) is necessarily dissipative. To obtain a complete and exact description of the full dynamics (of the closed system) we must embrace *all* correlation functions. One can actually construct a ‘master’ effective action for this purpose, as we now proceed to show.

### 3 The Master Effective Action

Our goal in this section is to show that it is (formally) possible to write down a functional of a c-number background field and a string of Green functions, such that variation of this functional yields the usual Dyson - Schwinger equations of QFT. For simplicity, we shall study a scalar field theory; since we are interested in the CTP formulation of the theory, however, the field must be thought of as carrying an internal index  $a$ , denoting the unprimed and primed quantities respectively, which is raised or lowered with a ‘metric’  $g_{ab} = \text{diag}(1, -1)$ . Also, CTP boundary conditions must be understood in all equations.

#### 3.1 Formal Construction of the Master Effective Action

We consider then a scalar field theory whose action

$$S[\Phi] = \frac{1}{2} S_2 \Phi^2 + S_{int}[\Phi] \quad (3.1)$$

decomposes into a free part and an interaction part

$$S_{int}[\Phi] = \sum_{n=3}^{\infty} \frac{1}{n!} S_n \Phi^n \quad (3.2)$$

Here and after, we use the shorthand

$$K_n \Phi^n \equiv \int d^d x_1 \dots d^d x_n K_{na^1 \dots a^n}(x_1, \dots x_n) \Phi^{a^1}(x_1) \dots \Phi^{a^n}(x_n) \quad (3.3)$$

where the kernel  $K$  is assumed to be totally symmetric.

Let us define also the ‘source action’

$$J[\Phi] = J_1 \Phi + \frac{1}{2} J_2 \Phi^2 + J_{int}[\Phi] \quad (3.4)$$

where  $J_{int}[\Phi]$  contains the higher order sources

$$J_{int}[\Phi] = \sum_{n=3}^{\infty} \frac{1}{n!} J_n \Phi^n \quad (3.5)$$

and define the generating functional

$$Z[\{J_n\}] = e^{iW[\{J_n\}]} = \int D\Phi e^{iS_t[\Phi, \{J_n\}]} \quad (3.6)$$

where

$$S_t[\Phi, \{J_n\}] = J_1 \Phi + \frac{1}{2} (S_2 + J_2) \Phi^2 + S_{int}[\Phi] + J_{int}[\Phi] \quad (3.7)$$

We shall also call

$$S_{int}[\Phi] + J_{int}[\Phi] = S_I \quad (3.8)$$

As it is well known, the Taylor expansion of  $Z$  with respect to  $J_1$  generates the expectation values of path - ordered products of fields

$$\frac{\delta^n Z}{\delta J_{1a^1}(x_1) \dots \delta J_{1a^n}(x_n)} = \langle P\{\Phi^{a^1}(x_1) \dots \Phi^{a^n}(x_n)\} \rangle \equiv F_n^{a^1 \dots a^n}(x_1, \dots x_n) \quad (3.9)$$

while the Taylor expansion of  $W$  generates the ‘connected’ Green functions (‘linked cluster theorem’ [53])

$$\frac{\delta^n W}{\delta J_{1a^1}(x_1) \dots \delta J_{1a^n}(x_n)} = \langle P\{\Phi^{a^1}(x_1) \dots \Phi^{a^n}(x_n)\} \rangle_c \equiv C_n^{a^1 \dots a^n}(x_1, \dots x_n) \quad (3.10)$$

Comparing these last two equations, we find the rule connecting the  $F$ ’s with the  $C$ ’s. First, we must decompose the ordered index set  $(i_1, \dots i_n)$  ( $i_k = (x_k, a^k)$ ) into all possible clusters  $P_n$ . A cluster is a partition of  $(i_1, \dots i_n)$  into  $N_{P_n}$  ordered subsets  $p = (j_1, \dots j_r)$ . Then

$$F_n^{i_1 \dots i_n} = \sum_{P_n} \prod_p C_r^{j_1 \dots j_r} \quad (3.11)$$

Now from the obvious identity

$$\frac{\delta Z}{\delta J_{ni_1 \dots i_n}} \equiv \frac{1}{n!} \frac{\delta^n Z}{\delta J_{i_1} \dots \delta J_{i_n}} \quad (3.12)$$

we obtain the chain of equations

$$\frac{\delta W}{\delta J_{ni_1 \dots i_n}} \equiv \frac{1}{n!} \sum_{P_n} \prod_p C_r^{j_1 \dots j_r} \quad (3.13)$$

We can invert these equations to express the sources as functionals of the connected Green functions, and define the master effective action (MEA) as the full Legendre transform of the connected generating functional

$$\Gamma_\infty[\{C_r\}] = W[\{J_n\}] - \sum_n \frac{1}{n!} J_n \sum_{P_n} \prod_p C_r \quad (3.14)$$

The physical case corresponds to the absence of external sources, whereby

$$\frac{\delta \Gamma_\infty[\{C_r\}]}{\delta C_s} = 0 \quad (3.15)$$

This hierarchy of equations is equivalent to the Dyson- Schwinger series.

### 3.2 The Master Effective Action and the Background Field Method

The master effective action just introduced becomes more manageable if one applies the background field method (BFM) [35] approach. We first distinguish the mean field and the two point functions

$$C_1^i \equiv \phi^i \quad (3.16)$$

$$C_2^{ij} \equiv G^{ij} \quad (3.17)$$

We then perform the Legendre transform in two steps: first with respect to  $\phi$  and  $G$  only, and then with respect to the rest of the Green functions. The first (partial) Legendre transform yields

$$\Gamma_\infty[\phi, G, \{C_r\}] \equiv \Gamma_2[\phi, G, \{j_n\}] - \sum_{n \geq 3} \frac{1}{n!} J_n \sum_{P_n} \prod_p C_r \quad (3.18)$$

Here  $\Gamma_2$  is the two particle-irreducible (2PI) effective action [3]

$$\Gamma_2[\phi, G, \{J_n\}] = S[\phi] + \frac{1}{2} G^{jk} S_{,jk} - \frac{i}{2} \ln \text{Det } G + J_{int}[\phi] + \frac{1}{2} G^{jk} J_{int,jk} + W_2 \quad (3.19)$$

and  $W_2$  is the sum of all 2PI vacuum bubbles of a theory whose action is

$$S'[\varphi] = \frac{i}{2}G^{-1}\varphi^2 + S_Q[\varphi] \quad (3.20)$$

$$S_Q[\varphi] = S_I[\phi + \varphi] - S_I[\phi] - S_I[\phi]_{,i} \varphi^i - \frac{1}{2}S_I[\Phi]_{,ij} \varphi^i \varphi^j \quad (3.21)$$

where  $\varphi$  is the fluctuation field around  $\phi$ , i.e.,  $\Phi = \phi + \varphi$ . Decomposing  $S_Q$  into source-free and source-dependent parts, and Taylor expanding with respect to  $\varphi$ , we may define the background-field dependent coupling and sources

$$S_Q[\varphi] = \sum_{n \geq 3} \frac{1}{n!} (\sigma_n + \chi_n) \varphi^n \quad (3.22)$$

where

$$\sigma_{ni_1 \dots i_n} = \sum_{m \geq n} \frac{1}{(m-n)!} S_{mi_1 \dots i_n j_{n+1} \dots j_m} \phi^{j_{n+1}} \dots \phi^{j_m} \quad (3.23)$$

$$\chi_{ni_1 \dots i_n} = \sum_{m \geq n} \frac{1}{(m-n)!} J_{mi_1 \dots i_n j_{n+1} \dots j_m} \phi^{j_{n+1}} \dots \phi^{j_m} \quad (3.24)$$

Now, from the properties of the Legendre transformation, we have, for  $n > 2$ ,

$$\frac{\delta W}{\delta J_n} \big|_{J_1, J_2} \equiv \frac{\delta \Gamma_\infty}{\delta J_n} \big|_{\phi, G} \quad (3.25)$$

Computing this second derivative explicitly, we conclude that

$$\frac{\delta W}{\delta J_n} \big|_{J_1, J_2} \equiv \frac{1}{n!} \phi^n + \frac{1}{2(n-2)!} G \phi^{n-2} + \sum_{m=3}^n \frac{\delta \chi_m}{\delta J_n} \frac{\delta W_2}{\delta \chi_m} \quad (3.26)$$

Comparing this equation with

$$\frac{\delta W}{\delta J_{ni_1 \dots i_n}} \equiv \frac{1}{n!} \sum_{P_n} \prod_p C_r^{j_1 \dots j_r} \quad (3.27)$$

we obtain the identity

$$\frac{\delta W_2}{\delta \chi_{ni_1 \dots i_n}} \equiv \frac{1}{n!} \sum_{P_n}^* \prod_p C_r^{j_1 \dots j_r} \quad (3.28)$$

where the  $*$  above the sum means that clusters containing one element subsets are deleted. This and the equality

$$\sum_{n \geq 3} \frac{1}{n!} J_n \sum_{P_n} \prod_p C_r = J_{int}[\phi] + \frac{1}{2} G^{ij} \frac{\delta J_{int}[\phi]}{\delta \phi^i \delta \phi^j} + \sum_{n \geq 3} \frac{1}{n!} \chi_n \sum_{P_n}^* \prod_p C_r \quad (3.29)$$

allow us to write

$$\begin{aligned} \Gamma_\infty[\phi, G, \{C_r\}] \equiv & S[\phi] + \left(\frac{1}{2}\right) G^{ij} \frac{\delta S[\phi]}{\delta \phi^i \delta \phi^j} - \frac{i}{2} \ln \text{Det } G \\ & + \{W_2[\phi, \{\chi_n\}] - \sum_{n \geq 3} \frac{1}{n!} \chi_n \sum_{P_n}^* \prod_p C_r\} \end{aligned} \quad (3.30)$$

This entails an enormous simplification, since it implies that to compute  $\Gamma_\infty$  it is enough to consider  $W_2$  as a functional of the  $\chi_n$ , without ever having to decompose these background dependent sources in terms of the original external sources.

### 3.3 Truncating the Master Effective Action: Loop Expansion and Correlation Order

After obtaining the formal expression for  $\Gamma_\infty$ , and thereby the formal hierarchy of Dyson - Schwinger equations, we should proceed with it much as with the BBGKY hierarchy in statistical mechanics [24], namely, truncate it and close the lower-order equations by constraining the high order correlation functions to be given (time-oriented) functionals of the lower correlations. Truncation proceeds by discarding the higher correlation functions and replacing them by given functionals of the lower ones, which represent the dynamics in some approximate sense [26]. The system which results is an open system and the dynamics becomes an effective dynamics.

It follows from the above that truncations will be generally related to approximation schemes. In field theory we have several such schemes available, such as the loop expansion, large  $N$  expansions, expansions in coupling constants, etc. For definiteness, we shall study the case of the loop expansion, although similar considerations will apply to any of the other schemes.

Taking then the concrete example of the loop expansion, we observe that the nonlocal  $\chi$  sources enter into  $W_2$  in as many nonlinear couplings of the fluctuation field  $\varphi$ . Now,  $W_2$  is given by a sum of connected vacuum bubbles, and any such graph satisfies the constraints

$$\sum n V_n = 2i \quad (3.31)$$

$$i - \sum V_n = l - 1 \quad (3.32)$$

where  $i, l, V_n$  are the number of internal lines, loops, and vertices with  $n$  lines, respectively. Therefore,

$$l = 1 + \sum \frac{n-2}{2} V_n \quad (3.33)$$

we conclude that  $\chi_n$  only enters the loop expansion of  $W_2$  at order  $n/2$ . At any given order  $l$ , we are effectively setting  $\chi_n \equiv 0$ ,  $n > 2l$ . Since  $W_2$  is a function of only  $\chi_3$  to  $\chi_{2l}$ , it



follows that the  $C_r$ 's cannot be all independent. Indeed, the equations relating sources to Green functions

$$\frac{\delta W_2}{\delta \chi_{ni_1 \dots i_n}} \equiv \frac{1}{n!} \sum_{P_n}^* \prod_p C_r^{j_1 \dots j_r} \quad (3.34)$$

have now turned, for  $n > 2l$ , into the algebraic constraints

$$\sum_{P_n}^* \prod_p C_r^{j_1 \dots j_r} \equiv 0 \quad (3.35)$$

In other words, the constraints which makes it possible to invert the transformation from sources to Green functions allow us to write the higher Green functions in terms of lower ones. In this way, we see that the loop expansion is by itself a truncation in the sense above and hence any finite loop or perturbation theory is intrinsically an effective theory.

Actually, the number of independent Green functions at a given number of loops is even smaller than  $2l$ . It follows from the above that  $W_2$  must be linear on  $\chi_n$  for  $l+2 \leq n \leq 2l$ . Therefore the corresponding derivatives of  $W_2$  are given functionals of the  $\chi_m$ ,  $m \leq l+1$ . Writing the lower sources in terms of the lower order Green functions, again we find a set of constraints on the Green functions, rather than new equations defining the relationship of sources to functions. These new constraints take the form

$$\sum_{P_n}^* \prod_p C_r^{j_1 \dots j_r} = f_n(G, C_3, \dots C_{l+1}) \quad (3.36)$$

for  $l+2 \leq n \leq 2l$ . In other words, to a given order  $l$  in the loop expansion, only  $\phi$ ,  $G$  and  $C_r$ ,  $3 \leq r \leq l+1$ , enter into  $\Gamma_\infty$  as independent variables. Higher correlations are expressed as functionals of these by virtue of the constraints implied by the loop expansion on the functional dependence of  $W_2$  on the sources.

However, these constraints are purely algebraic, and therefore do not define an arrow of time. The dynamics of this lower order functions is unitary. Irreversibility appears only when one makes a time-oriented ansatz in the form of the higher correlations, such as the 'weakening of correlations' principle invoked in the truncation of the BBGKY hierarchy [26]. This is done by substituting some of the allowed correlation functions at a given number of loops  $l$ , by solutions of the  $l$ -loop equations of motion. Observe that even if we use exact solutions, the end result is an irreversible theory, because the equations themselves are only an approximation to the true Dyson - Schwinger hierarchy.

To summarize, the truncation of the MEA in a loop expansion scheme proceeds in two stages. First, for a given accuracy  $l$ , an  $l$ -loop effective action is obtained which depends only on the lowest  $l+1$  correlation functions, say,  $\{\phi, G, C_3, \dots C_{l+1}\}$ . This truncated effective action generates the  $l$ -loop equations of motion for these correlation functions. In the second stage, these equations of motion are solved (with causal boundary conditions) for some of the correlation functions, say  $\{C_k, \dots C_{l+1}\}$ , and the result is substituted into the  $l$  loop effective action. (We say that  $\{C_k, \dots C_{l+1}\}$  have been *slaved* to  $\{\phi, G, C_3, \dots C_{k-1}\}$ ) The resulting

truncated effective action is generally complex and the mean field equations of motion it generates will come out to be dissipative, which indicates that the effective dynamics is stochastic.

We shall stop the formal development at this point and consider the relationship of these truncated ‘finite’ theories and the consistent histories approach to quantum mechanics, on which rests, in the final analysis, the physical interpretation of any quantum formalism. After that we will consider some familiar examples.

## 4 Truncated Effective Actions and Decoherence

The basic tenet of the consistent histories approach to quantum mechanics [14] is that quantum evolution may be considered as the result of the coherent superposition of virtual fine grained histories, each carrying full information on the state of the system at any given time. If we adopt the ‘natural’ procedure of specifying a fine grained history by defining the value of the field  $\Phi(x)$  at every space time point, these field values being c numbers, then the quantum mechanical amplitude for a given history is  $\Psi[\Phi] \sim e^{iS[\Phi]}$ , where  $S$  is the classical action evaluated at that particular history. The virtual nature of these histories is manifested through the occurrence of interference phenomena between pairs of histories. The strength of these effects is measured by the ‘decoherence functional’

$$D_F[\Phi, \Phi'] \sim \Psi[\Phi]\Psi[\Phi']^* \sim e^{i(S[\Phi]-S[\Phi'])} \quad (4.1)$$

In reality, actual observations correspond to ‘coarse grained’ histories. A coarse-grained history is defined in terms of a ‘filter function’  $\alpha$ , which determines which fine grained histories belong to the superposition, and their relative phases. For example, we may define a coarse-grained history of a system with two degrees of freedom  $x$  and  $y$  by specifying the values  $x_0(t)$  of  $x$  at all times. Then the filter function is  $\alpha[x, y] = \prod_{t \in R} \delta(x(t) - x_0(t))$ . The quantum mechanical amplitude for the coarse-grained history is defined as

$$\Psi[\alpha] = \int D\Phi e^{iS} \alpha[\Phi] \quad (4.2)$$

where the information on the quantum state of the field is assumed to have been included in the measure and/or the boundary conditions for the functional integral. The decoherence functional for two coarse-grained histories is [15]

$$D_F[\alpha, \alpha'] = \int D\Phi D\Phi' e^{i(S[\Phi]-S[\Phi'])} \alpha[\Phi] \alpha'[\Phi']^* \quad (4.3)$$

In this path integral expression, the two histories  $\Phi$  and  $\Phi'$  are not independent; they assume identical values on a  $t = T = \text{constant}$  surface in the far future. These are, of course, the same boundary conditions satisfied by the histories on each branch of the time path in the CTPEA discussed above.

To obtain a formulation of the consistent histories approach suitable for our present purpose, we shall (by appeal to Haag’s ‘reconstruction theorem’, which states that the set

of all expectation values of time - ordered products of fields carries full information about the state of the system [53]) consider fine grained histories as specified by the given values of the irreducible time - ordered correlation functions. These histories include those defined by the local value of the field, as those where all irreducible Feynman functions vanish, but allow also more general possibilities. Coarse grained histories will be specified by finite sets of Feynman functions, and correspond to the truncated theories described above.

In particular, consider two histories defined by two sets of mean fields, Feynman propagators and correlation functions up to  $l$  particles,  $\{\Phi, G, C_3, \dots C_l\}$  and  $\{\Phi', G', C'_3, \dots C'_l\}$ , respectively. Then the decoherence functional between them is given by [15]

$$D_F[\{\Phi, G, C_3, \dots C_l\}, \{\Phi', G', C'_3, \dots C'_l\}] \sim e^{i\Gamma_l} \quad (4.4)$$

where  $\Gamma_l$  is the  $l$ -loop CTP effective action evaluated at the following history: (Here we have disregarded a prefactor which is not important to our discussion.)

- a) Correlation functions on the ‘direct’ branch are defined according to the first history:  $\Phi = \phi$ ,  $G^{11} = G$ , etc.
- b) Those on the ‘return’ branch are identified with the time-reverse of those in the second history:  $\Phi' = \phi'$ ,  $G^{22} = (G')^*$ , etc.
- c) All others are slaved to these.

We have already discussed in detail the rationale of the ansatz (4.4) [15]. This ansatz generalizes the decoherence functional (4.1) between histories defined in the usual way, in that they reduce to it when all irreducible Green functions are chosen to vanish, and it is consistent, in the sense that further integration over the higher correlation functions  $\{C_{k+1}, \dots C_l\}$ , say, gives back the decoherence functional appropriate to the  $k$  loop theory in the saddle point approximation to the trace.

In the spirit of the closed-time path method we might wish to consider more general histories, where not only time-ordered products, but also others are freely specified. However, this cannot be done in the present framework, because we are describing each of the two histories entering into the decoherence functional as a  $c$ -number field distribution defined on a simple time path. Thus, we cannot define different composite operators. Whenever we write down a product of fields at different points, it is automatically time ordered by the path integral. We could achieve our goal, notwithstanding, if the fine grained histories were themselves defined on a closed time path, thus lifting the restriction to time ordered products. We shall not consider here these extensions of the theory.

In actual practice, we are not even interested in exhausting the possible accuracy at a given number of loops, but rather in how further higher correlation functions are slaved to the lower ones. The relevant effective action under the ansatz (4.4) then becomes the truncated effective action we discussed above. Because the truncated effective action has in general a positive imaginary part, this implies a suppression of the overlap between different truncated histories, or decoherence.

Decoherence means physically that the different coarse grained histories making up the full quantum evolution are individually realizable and may thus be assigned definite probabilities in the classical sense. Therefore, the quantum nature of the system will be shredded

to the degree of accuracy afforded by the coarse graining procedure, and the dynamics be described by a self-consistent, coupled set of equations of a finite number of (non local) c-number variables.

In this finite, truncated theory, decoherence is associated to information degradation and loss of full predictability [20]. This dynamics is, however, non-deterministic and contains necessarily a stochastic component. We shall see here that it is a consequence, within the consistent histories language, of the categorical relationship between dissipation and noise embodied in the fluctuation dissipation relation [12].

At this point it is perhaps useful to look at some concrete examples. We shall consider the stochastic dynamics of respectively the mean field and the Feynman functions of a self interacting scalar field.

## 4.1 Decoherence, Dissipation and Noise in $\lambda\Phi^4$ theory

Let us now look at the example of a self interacting scalar field theory with classical action

$$S[\Phi] = \int d^4x \left\{ -\frac{1}{2} \left( \partial^\mu \Phi \partial_\mu \Phi + m^2 \Phi^2 \right) - \frac{\lambda}{4!} \Phi^4 \right\} \quad (4.5)$$

and consider fluctuations of the mean field around the symmetric state. We shall first analyze the theory at one and two loop order. In the next section we shall then adopt the new ‘correlation histories’ viewpoint [15], which will require consideration of three loop effects.

### 4.1.1 Dynamics of the mean field: one loop analysis

It should be clear from the above that the one loop expansion has very special features. To this order all  $\chi$ ’s vanish, so only the background field  $\phi$  and the two-point functions  $G$  remain, and there is no Legendre transformation to perform. We find

$$\Gamma_\infty[\phi, G, \{C_r\}] \rightarrow \Gamma_1[\phi, G] \equiv S[\phi] + \frac{1}{2} G^{ij} \frac{\delta S[\phi]}{\delta \phi^i \delta \phi^j} - \frac{i}{2} \ln \text{Det } G \quad (4.6)$$

By symmetry, there is a solution of the semiclassical equations with  $\phi \equiv 0$  and

$$S_{2jk} - iG_{jk}^{-1} = 0 \quad (4.7)$$

Writing (signature -+++)  $S_{2jk} = (\square - m^2)g_{jk}$ , and taking into account the CTP boundary conditions, we get via

$$G^{ab}(x, x') = \int \frac{d^d k}{(2\pi)^d} e^{-ik(x-x')} G^{ab}(k) \quad (4.8)$$

the usual result [46]

$$\begin{aligned} G^{11} &= (G^{22})^* = (-i)(k^2 + m^2 - i\epsilon)^{-1} \\ G^{12}(k) &= G^{21}(-k) = 2\pi\theta(k^0)\delta(k^2 + m^2) \end{aligned} \quad (4.9)$$

We now look for the dynamics (and coherence properties) of fluctuations around the zero point. Again, because of the field-reversal symmetry, we know the fluctuations in the background field cannot couple to the fluctuations in the propagators (to quadratic order). For simplicity, we shall omit these, and consider only a perturbation  $\Delta\phi$  on the background field. The quadratic part of the effective action becomes

$$\Delta\Gamma_1[\Delta\phi] \equiv \frac{1}{2}\Delta\phi^j[(\square - m^2)g_{jk} - \lambda_{jklm}G^{lm}]\Delta\phi^k \quad (4.10)$$

where

$$\lambda_{ijkl} = \pm\lambda\delta(x_j - x_i)\delta(x_k - x_i)\delta(x_l - x_i) \quad (4.11)$$

for  $a_i = a_j = a_k = a_l = 1, 2$  respectively for the plus and minus signs, and  $\lambda_{ijkl} = 0$  otherwise. Since only the coincidence limit of the propagators is involved, the effective action is real, and so we reach our first conclusion: *There is no quantum to classical transition for fluctuations around the symmetric vacuum at one loop order, and, in particular, no such transition for free field theories*<sup>2</sup>.

#### 4.1.2 Dynamics of the mean field: two loops analysis

As discussed above, two loops is the first order where a nontrivial truncation is called for. At this order, we have two graphs contributing to  $W_2$

$$W_2 = \frac{i}{12}(\sigma_3 + \chi_3)_{ijk}G^{ii'}G^{jj'}G^{kk'}(\sigma_3 + \chi_3)_{i'j'k'} + \frac{1}{8}(\sigma_4 + \chi_4)_{ijkl}G^{ij}G^{kl} \quad (4.12)$$

Observe that with the present conventions,  $\sigma_{4ijkl} = -\lambda_{ijkl}$ , so the second graph agrees with (3.6) in [1]. As discussed above, the variation of  $W_2$  with respect to  $\chi_4$  yields the (time symmetric) constraint  $C_4 \equiv 0$ . Moreover, since  $\chi_4$  enters linearly in  $W_2$ , it disappears in the Legendre transform. So the only nontrivial correlation, besides the  $G$ s, is  $C_3$ . Now, variation with respect to  $\chi_3$  yields

$$C_3^{ijk} = iG^{ii'}G^{jj'}G^{kk'}(\sigma_3 + \chi_3)_{i'j'k'} \quad (4.13)$$

Solving for  $\chi_3$ , we immediately find the two-loops effective action

$$\begin{aligned} \Gamma_2 \equiv & S[\phi] + \frac{1}{2}G^{ij}\frac{\delta S[\phi]}{\delta\phi^i\delta\phi^j} - \frac{i}{2}\ln \text{Det } G \\ & + \frac{i}{12}C_3^{ijk}G_{ii'}^{-1}G_{jj'}^{-1}G_{kk'}^{-1}C_3^{i'j'k'} + \frac{1}{6}\sigma_{3ijk}C_3^{ijk} + \frac{1}{8}\sigma_{4ijkl}G^{ij}G^{kl} \end{aligned} \quad (4.14)$$

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<sup>2</sup>An unrelenting critic could observe that there is an imaginary part in the one loop effective action, since we need to add the usual  $i\epsilon$ 's to the mass to enforce the vacuum initial conditions. Since the limit  $\epsilon \rightarrow 0$  has to be taken anyway, the resulting noise is infinitely weak. It only means that a 'stable' particle with infinite lifetime is only an idealized conception. As observed by Schwinger, any physically observable particle has necessarily a finite lifetime [54]

As with one loop order, symmetry implies the existence of a solution with  $\phi = C_3 = 0$ , and

$$S_{2jk} - iG_{jk}^{-1} - \frac{1}{2}\lambda_{jklm}G^{lm} = 0 \quad (4.15)$$

The modification with respect to the one loop equation amounts to the resummation of daisy graphs. Observe that these equations remain time-reversal invariant. Indeed, we could assume that  $m^2$  is already the physical mass, so the tadpole graph vanishes, and the propagators would remain the same as before.

The difference between one and two loops physics appears when we look at fluctuations. Indeed, we can still assume that the propagators remain unperturbed; however, the perturbation  $\Delta\phi^i$  to the background field now couples to the perturbation  $\Delta C_3$  to the three point correlation. The quadratic action now reads

$$\begin{aligned} \Delta\Gamma_2 \equiv & \frac{1}{2}\Delta\phi^j[(\square - m^2)g_{jk} - \lambda_{jklm}G^{lm}]\Delta\phi^k \\ & + \frac{i}{12}\Delta C_3^{ijk}G_{ii'}^{-1}G_{jj'}^{-1}G_{kk'}^{-1}\Delta C_3^{i'j'k'} - \frac{1}{6}\lambda_{ijkl}\Delta\phi^l\Delta C_3^{ijk} \end{aligned} \quad (4.16)$$

Still there is no sign of dissipation or loss of coherence. However, we do get the nontrivial ‘mean field’ equation

$$iG_{ii'}^{-1}G_{jj'}^{-1}G_{kk'}^{-1}\Delta C_3^{i'j'k'} - \lambda_{ijkl}\Delta\phi^l = 0 \quad (4.17)$$

for the three point function. If one imposes the time-oriented ansatz

$$\Delta C_3^{ijk} = -iG^{ii'}G^{jj'}G^{kk'}\lambda_{i'j'k'l'}\Delta\phi^{l'} \quad (4.18)$$

and substitute into the effective action, we obtain the truncated effective action

$$\Delta\Gamma_{2T}[\Delta\phi] \equiv \frac{1}{2}\Delta\phi^j((\square - m^2)g_{jk} - \lambda_{jklm}G^{lm})\Delta\phi^k + \frac{i}{12}\Delta\phi^j\Sigma_{jk}\Delta\phi^k \quad (4.19)$$

where  $\Sigma_{jk}$  represents the setting sun graph

$$\Sigma_{ll'} = \lambda_{ijkl}G^{ii'}G^{jj'}G^{kk'}\lambda_{i'j'k'l'}$$

Our ansatz represents, of course, the slaving of the fluctuations in the correlations to those in the background field. Moreover, in the physical limit where  $\phi = \phi'$ , the relationship is causal. In other words, we ascribe any deviation from Gaussian correlations to the presence of quanta emitted by the fluctuating background field.

The new term in the effective action can be written in the form (2.15) [11], where

$$\mathcal{D}(x, x') = [-\square + m^2 + \lambda G_F(x, x)]\delta(x - x') + D(x, x'), \quad (4.20)$$

where

$$\begin{aligned} D(x, x') &= \frac{\lambda^2}{3} [\text{Im}(G^{11}(x, x'))^3] \theta(t - t') \\ N(x, x') &= \frac{\lambda^2}{6} [\text{Re}(G^{11}(x, x'))^3] \end{aligned} \quad (4.21)$$

which makes manifest the real and imaginary parts. The relevant graph has been computed in detail in [1] (Eq. 4.27). However, there is a more direct way to appreciate the physical meaning of these kernels. From Poincaré invariance and the time - ordered property we must have

$$[G^{11}(x, x')]^3 = \frac{-i\mu^{2\epsilon}}{(4\pi)^4} \int \frac{d^d k}{(2\pi)^d} e^{-ik(x-x')} F(-k^2 + i\epsilon) \quad (4.22)$$

where  $F$  itself admits the Lehmann representation [55]

$$F(z) = a + b(z - m^2) + (z - m^2)^2 \int_{9m^2}^{\infty} \frac{ds h(s)}{(s - z)(s - m^2)^2} \quad (4.23)$$

where  $a$  and  $b$  are actually divergent

$$\begin{aligned} a &= 3m^2 \left[ \frac{2}{\epsilon^2} + \frac{2}{\epsilon} (A + B \ln \frac{m^2}{4\pi\mu^2}) + \ln^2 \frac{m^2}{4\pi\mu^2} + C \ln \frac{m^2}{4\pi\mu^2} + D \right] \\ b &= \frac{1}{2\epsilon} + \frac{1}{2} \ln \frac{m^2}{4\pi\mu^2} + E \end{aligned} \quad (4.24)$$

Here,  $A, B, C, D, E$  are numerical constants, and  $\epsilon = d - 4$  [1]. These terms may be absorbed into the classical action and need not worry us. More relevant to our concern is the  $h$  function, which gives the imaginary part of  $F$ . It must be positive, because of the optical theorem. An actual evaluation yields

$$h(s) = \frac{16}{s} \int_{4m^2}^{(\sqrt{s}-m)^2} dt \sqrt{(s + m^2 - t)^2 - 4sm^2} \sqrt{1 - \frac{4m^2}{t}} \theta(s - 9m^2) \quad (4.25)$$

With this knowledge, it is immediate to find the Fourier expansions

$$\begin{aligned} D(x, x') &= \frac{\lambda^2}{6} \frac{\mu^{2\epsilon}}{(4\pi)^4} \int \frac{d^d k}{(2\pi)^d} e^{-ik(x-x')} \int_{9m^2}^{\infty} \frac{ds h(s)}{(s + (k + i\epsilon)^2)} \\ N(x, x') &= \frac{\lambda^2}{6} \frac{\mu^{2\epsilon}}{(4\pi)^4} \int \frac{d^d k}{(2\pi)^d} e^{-ik(x-x')} \pi h(-k^2) \theta(-k^2 - 9m^2) \end{aligned} \quad (4.26)$$

where  $(k + i\epsilon)^2 = -(k_0 + i\epsilon)^2 + \vec{k}^2$ . Associating as usual the imaginary part of the effective action with noise and decoherence, and the imaginary part (in momentum space) of the retarded propagator with dissipation, we conclude at once that: *Only the amplitudes*

corresponding to momenta above the three particle threshold lose their quantum coherence and become classical. Therefore, only those amplitudes are subject to random fluctuations in the semiclassical regime. The corresponding statement in terms of dissipation would be: *Only the amplitudes corresponding to momenta above the three particle threshold are subject to dissipation.*

The presence of a fluctuation - dissipation relation underlying all this should be obvious, but we can be more explicit still. We know that the noise affecting the above- threshold amplitudes will have a mean square value

$$\sigma^2(k) \sim N(k) \sim \frac{\lambda^2}{6} \frac{\mu^{2\epsilon}}{(4\pi)^4} \pi h(-k^2) \quad (4.27)$$

The FDR is then just the statement that

$$\text{Im } G_{ret} \sim |G_{ret}|^2 N(k) \quad \text{when } -k^2 > 9m^2, \quad (4.28)$$

thus linking the noise autocorrelation to the spectrum of dissipated energy, just as in the early formulations by Callen and Welton and as we have illustrated in the cosmological particle creation problem [46, 47]. Callen and Welton's formulation of the theorem is recovered by identifying

$$Z = (i\omega G_{ret})^{-1}, \quad R = \text{Re}Z = \left(\frac{1}{\omega}\right) N(k) \quad (4.29)$$

So, given  $\vec{k}$ , we find

$$\sigma^2 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} N(k) = \frac{2}{\pi} \int_0^{\infty} d\omega \left(\frac{\omega}{2}\right) R(\omega) \quad (4.30)$$

as expected.

This leads to an important point, namely that one should not be too quick in identifying the Hadamard function of the quantum field with the correlator of the stochastic field. In our case, the quantum Hadamard function also has an on-shell contribution; however, this part does not appear in the classical correlator, as on shell fluctuations are dissipation free, and thus never become classical. Another popular application is galaxy formation seeded by quantum fluctuations in inflationary cosmology. The conventional wisdom is to identify straightforwardly the classical correlator with the full Hadamard kernel, rather than only to the part partaking in decoherence, as we assert here. The conventional way leads to a gross overestimation of the amplitude of the resulting inhomogeneities [48].

This distinction between on and off shell fluctuations disappears in the important case of massless fields. In this case, we get  $h(-k^2) \sim -8k^2$  for every time-like 4-momentum. Still, different components of the field will have different characteristic times at which they become classical.



## 5 Correlation Noise

While describing the dynamics of a quantum field in terms of the mean field or order parameter is the most familiar approach to causal field theory, in many applications the relevant information is most readily retrieved not from the mean field, but from the correlation functions [42]. Moreover, as we have seen above, the sequence of correlations introduces in non linear field theory an intrinsic hierarchical ordering, which allows us to fill the interpolating steps between the complete unitary underlying field theory and the truncated effective descriptions that account for our actual experience, without imposing any *ad hoc* concept of relevance or distinction from outside. As a demonstration of this more sophisticated approach to decoherence and noise in field theory, in this section we shall continue to work with the  $\lambda\Phi^4$  theory as example. Here we shall simply assume that the mean field vanishes identically (as it usually does, because of symmetry) and focus directly on the dynamics of the two point functions.

### 5.1 Naive derivation of ‘correlation noise’

As it turns out, the case in point is simple enough to deduce the Langevin type of Schwinger-Dyson equation for the propagators by an *ad hoc* argument, totally independent of the MEA formalism. It is instructive to review this argument, since it sheds light on the physics involved, and will serve as a check on our formulation when we proceed with the formal development below.

We start with the Heisenberg equation of motion for the field operator

$$\left(\square - m^2\right) \Phi(x) - \frac{\lambda}{6} \Phi^3(x) = 0 \quad (5.1)$$

and the canonical commutation relation

$$\left[\frac{\partial \Phi}{\partial t}(t, \vec{x}_1), \Phi(t, \vec{x}_2)\right] = -i\delta(\vec{x}_1 - \vec{x}_2) \quad (5.2)$$

Let us introduce the time ordered product of fields

$$T[\Phi(x_1)\Phi(x_2)] = \theta(t_1 - t_2)\Phi(x_1)\Phi(x_2) + \theta(t_2 - t_1)\Phi(x_2)\Phi(x_1) \quad (5.3)$$

From (5.1) and (5.2) we derive the operator equation

$$\left(\square_1 - m^2\right) T[\Phi(x_1)\Phi(x_2)] - \frac{\lambda}{6} T[\Phi^3(x_1)\Phi(x_2)] = i\delta(x_1 - x_2) \quad (5.4)$$

In the present context, a correlation history is defined by selecting those field histories where  $T[\Phi^i\Phi^j]$  remains ‘close’ to a given kernel  $G_F$ . If Wick’s theorem holds, then we could write also the second term in (5.4) in terms of binary field products, as

$$\begin{aligned}
T \left[ \Phi^3(x_1) \Phi(x_2) \right] &\sim 3G_F(x_1, x_1) G_F(x_1, x_2) \\
&- i\lambda \int d^4y [G_F^3(x_1, y) G_F(y, x_2) - G^{-3}(x_1, y) G^-(y, x_2)] + \dots
\end{aligned} \tag{5.5}$$

Here we are not writing explicitly the one- particle or higher order reducible graphs,  $G^-$  stands for the ‘value ’ of the composite operator  $\Phi(x_1)\Phi(x_2)$  in the given correlation history. Since Wick’s formula is not exact, however, the best we can do is to write

$$\begin{aligned}
&(\square_1 - m^2) G_F(x_1, x_2) - \frac{1}{2}\lambda G_F(x_1, x_1) G_F(x_1, x_2) \\
&+ \frac{i}{6}\lambda^2 \int d^4y [G_F^3(x_1, y) G_F(y, x_2) - G^{-3}(x_1, y) G^-(y, x_2)] + \dots \\
&= i\delta(x_1 - x_2) + \frac{1}{2}\lambda f(x_1, x_2)
\end{aligned} \tag{5.6}$$

where

$$f(x_1, x_2) \sim \frac{1}{3}T \left[ \Phi^3(x_1) \Phi(x_2) \right] - G_F(x_1, x_1) G_F(x_1, x_2) \tag{5.7}$$

is ‘small ’. The equation for  $G_F$  simplifies substantially if we operate on it from the left with  $G_F^{-1} \sim -i(\square - m^2)$ . We get

$$\begin{aligned}
-iG_F^{-1}(x_1, x_2) + (\square - m^2) \delta(x_1, x_2) &- \frac{1}{2}\lambda G_F(x_1, x_1) \delta(x_1, x_2) + \frac{i}{6}\lambda^2 G_F^3(x_1, x_2) \\
&= \frac{1}{2}\lambda K(x_1, x_2) \delta(x_1, x_2),
\end{aligned} \tag{5.8}$$

where now all the terms up to order  $\lambda^2$  are explicit, and the driving force is

$$K(x_1, x_2) \sim \{ \Phi(x_1) \Phi(x_2) - G_F(x_1, x_2) \}. \tag{5.9}$$

Of course,  $K$  is a q-number quantity , which reminds us that  $G_F$  is a composite operator. However, after decoherence, we can look upon  $G_F$  as the actual value of the time ordered product *and* a c-number quantity. In the semiclassical regime,  $K$  becomes a *bona fide* classical stochastic force. In a first approximation, we can take this force to be Gaussian, with a zero mean. Following the usual procedure [23], we derive its standard deviation from the symmetric expectation value

$$\langle K(x_1, x_2) K^*(y_1, y_2) \rangle_c \equiv \frac{1}{2} \left\langle \left\{ K(x_1, x_2), K^\dagger(y_1, y_2) \right\} \right\rangle_q \tag{5.10}$$

to be

$$\langle K(x_1, x_2) K^*(y_1, y_2) \rangle_c = \frac{1}{2} \left\{ G^+(x_1, y_1) G^+(x_2, y_2) + G^-(x_1, y_1) G^-(x_2, y_2) + (y_1 \rightleftharpoons y_2) \right\} \quad (5.11)$$

The ‘naive’ derivation of the Langevin type Schwinger - Dyson equation may be carried through directly at the level of fields defined on a closed time path. We only quote the result

$$-\frac{i}{2} (G^{-1})_{ii'} + \frac{1}{2} g_{ii'} (\square - m^2) - \frac{1}{4} \lambda_{ii'jj'} G^{jj'} + \frac{i}{12} \lambda_{ijkl} G^{jj'} G^{kk'} G^{ll'} \lambda_{i'j'k'l'} = \frac{1}{2} \lambda_{ii'jj'} K^{jj'} \quad (5.12)$$

where

$$K^{ij} \sim \Phi^i \Phi^j - G^{ij} \quad (5.13)$$

Upon decoherence  $K$  becomes a classical stochastic source. Observe that the (12) and (21) equations are not modified; also that  $K$  is generally a complex source, with  $K^{22} = (K^{11})^*$ . We recover a positive definite noise autocorrelator as

$$\langle K^{11} K^{22} \rangle_c = \langle K(x_1, x_2) K^*(y_1, y_2) \rangle_c \quad (5.14)$$

The right hand side is given, again, by (5.11). This is the ‘naive’ derivation of the Langevin equation for the Feynman propagator.

## 5.2 Systematic study of ‘correlation noise’

We shall now show how to recover the results of our ‘naive’ approach to the stochastic behavior of Green functions, by systematically following the theory sketched in the previous Sections.

We need to carry out our analysis at three loops order, this being the lowest order at which the dynamics of the correlations is nontrivial, in the absence of a symmetry breaking background field [1]. To this accuracy, we have room for four nonlocal sources besides the mean field and the two point correlations, namely  $\chi_3, \chi_4, \chi_5$ , and  $\chi_6$ . However, the last two enter linearly in the generating functional. Thus the three- loop effective action only depends nontrivially on the mean field and the two, three and four point correlations. By symmetry, there must be a solution where the mean field and the three point function remain identically zero, which we shall assume. Slaving the four point function to the propagators, we find the truncated action [1]

$$\Gamma_3[G] \equiv \frac{1}{2} g_{ij} (\square - m^2) G^{ij} - \frac{i}{2} \ln \text{Det } G - \frac{1}{8} \lambda_{ii'jj'} G^{ii'} G^{jj'} + \frac{i}{48} \lambda_{ijkl} G^{ii'} G^{jj'} G^{kk'} G^{ll'} \lambda_{i'j'k'l'} \quad (5.15)$$

and the mean field equation of motion for the propagators

$$-\frac{i}{2} (G^{-1})_{ii'} + \frac{1}{2} g_{ii'} (\square - m^2) - \frac{1}{4} \lambda_{ii'jj'} G^{jj'} + \frac{i}{12} \lambda_{ijkl} G^{jj'} G^{kk'} G^{ll'} \lambda_{i'j'k'l'} = 0 \quad (5.16)$$

which reduces to the left hand side of (5.12) .

To derive the right hand side of the Langevin type Dyson-Schwinger equation, we must consider fluctuations around the mean field solution. Concretely, we expect to retrieve the noise autocorrelation (5.9) from the second variation of the two particle irreducible (2PI) effective action, evaluated at the extremum point. A perturbation  $\Delta^{ij}$  to the propagators  $G^{ij}$  will satisfy the equations

$$\frac{i}{2} (G^{-1})_{ij} \Delta^{jj'} (G^{-1})_{j'i'} - \frac{1}{4} \lambda_{ii'jj'} \Delta^{jj'} + \frac{i}{4} \lambda_{ijkl} G^{jj'} G^{kk'} \Delta^{ll'} \lambda_{i'j'k'l'} = 0 \quad (5.17)$$

The linearized equations for the perturbations follow from the quadratic 2PI effective action [3] [1]

$$\Delta\Gamma = \frac{i}{4} \text{Tr} \left[ (G^{-1})_{ij} \Delta^{jj'} \right]^2 - \frac{1}{8} \lambda_{ii'jj'} \Delta^{ii'} \Delta^{jj'} + \frac{i}{8} \lambda_{ijkl} \Delta^{ii'} G^{jj'} G^{kk'} \Delta^{ll'} \lambda_{i'j'k'l'} \quad (5.18)$$

The connection between the effective action and the noise follows from the observation that the decoherence functional between a correlation history with the Feynman propagator, say,  $G_F + \Delta^{11}$ , and another with  $G_F + (\Delta^{22})^*$  should be given by  $D_F \sim \exp(i\Delta\Gamma)$ . However, in this case we cannot directly identify  $\Delta\Gamma$  from (5.18) as the ‘phase’ of the decoherence functional. This is obvious already from the fact that the decoherence functional depends only on two kernels  $\Delta^{11}$  and  $(\Delta^{22})^*$ , while  $\Delta\Gamma$  depends on four kernels, both the above and also the positive and negative frequency propagators  $\Delta^{21}$  and  $\Delta^{12}$ . In order to retrieve the decoherence functional from the quadratic effective action, we must first ‘slave’ these excess kernels to the time ordered ones. We do this by extremizing  $\Delta\Gamma$  with respect to  $\Delta^{12,21}$ , holding  $\Delta^{11,22}$  fixed. Only when we substitute the slaved propagators back into the effective action can we establish the connection to decoherence and noise.

Because of the inherent complexity of the 2PI effective action, it is necessary to impose certain restrictions on the variations allowed on the propagators, in order to arrive at meaningful results. We shall consider only real perturbations of the propagators, and assume that the linearized equations hold at  $O(\lambda)$  (Note that in [15] a different set of restrictions were imposed, so the results below are correspondingly different.)

As in the previous section, it is convenient to switch to variables describing the propagators on the ‘diagonal’ of the decoherence functional, that is, when both histories are the same, and the departure from this diagonal. Since on the diagonal we have  $\Delta^{11} = \Delta^{22}$  it is convenient to define

$$\{\Delta\} = \Delta^{11} + \Delta^{22}, \quad [\Delta] = \Delta^{11} - \Delta^{22} \quad (5.19)$$

According to the CTP boundary conditions,  $\Delta^{12}(x, x')$  should turn into the Dyson function as  $x$  “goes round the corner” at  $T \rightarrow \infty$ , and similarly  $\Delta^{21}$  turns into the Feynman function. This dictates the boundary conditions at infinity, and since all propagators involved are Klein Gordon solutions at tree level, this means that

$$\Delta^{21}(x, x') \equiv \Delta^{11}(x, x'); \quad \Delta^{12}(x, x') \equiv \Delta^{22}(x, x') \quad (5.20)$$

throughout, at tree level. The  $O(\lambda^2)$  terms which have the generic form

$$i\lambda^2 \int d^4x d^4x' \left\{ G_F^2(x, x')(\Delta^{11})^2(x, x') - G_+^2(x, x')(\Delta^{21})^2(x, x') \right. \\ \left. - G_-^2(x, x')(\Delta^{12})^2(x, x') + G_D^2(x, x')(\Delta^{22})^2(x, x') \right\} \quad (5.21)$$

now become

$$i\lambda^2 \int d^4x d^4x' \left\{ \theta(t' - t) [G_-^2(x, x') - G_+^2(x, x')] [(\Delta^{11})^2 - (\Delta^{22})^2](x, x') \right\} \quad (5.22)$$

They are manifestly real, and are thus unrelated to noise.

Finally, observe that the linearized equations imply, at  $O(\lambda)$ , the identity

$$[G^{-1}\Delta]_i^j(x, y) = -\frac{i}{2}\lambda \begin{pmatrix} \Delta^{11}(x, x) G^{11}(x, y) & \Delta^{11}(x, x) G^{12}(x, y) \\ -\Delta^{22}(x, x) G^{21}(x, y) & -\Delta^{22}(x, x) G^{22}(x, y) \end{pmatrix} \quad (5.23)$$

Computing the trace of the square of this matrix, and substituting into the linearized effective action, we recover its imaginary part as

$$\text{Im}[\Delta\Gamma] = -\frac{1}{32}\lambda^2 \int d^4x d^4y \left\{ [\Delta](x, x) \left[ (G^+(x, y))^2 + (G^-(x, y))^2 \right] \right\} [\Delta](y, y) \quad (5.24)$$

The ‘wrong’ sign reminds us that the stochastic forces are generally complex. However, the correlator

$$\langle KK^* \rangle \sim - \left( \frac{\partial^2}{\partial [\Delta]^2} \right) \text{Im}[\Delta\Gamma] \quad (5.25)$$

comes out positive, as it should. This gives the same result as from our earlier ‘naive’ derivation.

## 6 Discussions

In this paper we have presented a new approach to the analysis of the statistical mechanics of interacting quantum field theory based on the properties and dynamics of the infinite number of correlation functions of the field. We point out that ordinary perturbative field theory involving only a finite number of correlation functions amounts to an open system obtained by truncating the Dyson- Schwinger equations for the hierarchy of correlations. Inclusion

of the effect of the higher correlation functions leads to modifications of the dynamics of the lower order functions appearing as fluctuations, noise, dissipation and irreversibility. This proves our earlier conjecture [1, 4, 22], that any effective field theory is intrinsically dissipative and stochastic in nature.

Our approach is inspired by the Boltzmann-BBGKY formulation of the kinetic theory of dilute gases [26, 27], and incorporates the ideas of noise and fluctuations in the Langevin-Fokker-Planck approach to the stochastic mechanics of Brownian motion [29]. The techniques we used are the closed time path [2] and the multiple source effective actions [3]. Decoherence and quantum to classical transition are discussed in the consistent history conceptual framework. We see once again [16, 17] that decoherence, dissipation and noise are indissolubly related to each other in any nontrivial interacting theory. Therefore the best classical description of the dynamics is both irreversible and noisy. If the description is framed in terms of correlations rather than the more familiar mean fields, then the propagators themselves will become stochastic, generating the so called ‘correlation noises’.

Possible applications of the viewpoint expounded and the results derived in this paper are manifold, as statistical ideas and measures such as extraction of relevant information, formulation in terms of collective variables, truncation and coarse graining of irrelevant variables permeate almost all areas of theoretical physics where one wants to derive the macroscopic features of a system in terms of microscopic laws or when one attempts to give a simple description to complex phenomena. As guides to our present investigation, in addition to the paradigmatic schemes of non-equilibrium statistical mechanics and the philosophical underpinnings of decoherent histories approach to quantum mechanics, it is worth mentioning that as concrete problems with deep meanings, *critical dynamics* provides not only a physical model for our thoughts but also a challenge to solving some of its outstanding problems. For example, there is at present no satisfactory understanding of nonequilibrium first order phase transitions, as most work on the subject rests on Euclidean methods which presuppose equilibrium conditions. Following the pioneering work of Calzetta [42] there have been more recent work to apply the CTP method to the study of phase transitions in the early universe [56]. Our present formalism can provide a more rigorous foundation for these investigations.

Fluctuation phenomena giving rise to instability of structure and growth [57, 58] is a new and interesting direction of potential development and application. In this connection, an active area where our approach will be directly applicable is *structure formation* from primordial fluctuations in the early universe. It is believed that these (classical) primordial fluctuations evolved out of quantum fluctuations of the matter fields, got amplified in the inflationary stage of the universe [59, 60], and became classical as they grew larger than the Hubble radius. Most existing treatments pay no attention to the subtle and important issue of quantum to classical transition, i.e., transition from a pure quantum to a stochastic classical description of the fluctuations. These unsatisfactory aspects of conventional wisdom have been pointed out by Hu, Paz and Zhang [21], who outlined a stochastic field theoretical derivation of the noise and fluctuations from two interacting fields. The present authors [11] have also pointed out a fundamental flaw in the conventional treatment of classical fluctua-

tions, vis., in simply identifying them with the Hadamard functions. Detailed discussions of this issue from the perspective of this paper can be found in [48].

We have already mentioned the direct implication of our viewpoint and results on *effective field theory* [22]. How successful a given theory can describe the physical world really depends on the relevant (energy, space, or time) scales in question and the degree of precision (or power of resolution) of measurements. A theory can be ‘perfect’ at a low energy scale but makes no sense at all at a higher energy scale. How a fundamental theory appears at low energies or long wavelengths (infrared behavior) is a question often asked, from gauge hierarchy and dimensional reduction in particle physics to the approach to critical point in phase transitions. How can one infer the attributes of the ‘fundamental’ theory at high energies from our limited knowledge at low energies? This is admittedly a more difficult question, but not exactly a futile one. It has been the route taken in our search from atomic to nuclear to particle physics, and today, towards unified theories including quantum gravity. Understanding the nature of effective theories is, in our opinion, essential for grasping the interconnection of physical phenomena at different scales and interaction ranges, and for probing into uncharted domains. For example, it has been speculated that near the Planck time, spacetime fluctuations will grow and the smooth manifold picture in the semiclassical approximation will give way to stochastic behavior before the quantum gravity regime fully takes over [11, 9, 12]. The Langevin- type equations for lower correlation functions driven by the correlation noise in our framework can be used as models to examine possible *phase transition behavior from the semiclassical to the quantum regime* at the Planck time.

The other long-standing motivation for us to work out the statistical mechanical properties of quantum fields is to analyze the *black hole entropy and information loss problem*. We feel that our current understanding of the black hole backreaction and entropy problems based on equilibrium thermodynamics is inadequate. To tackle the collapse and backreaction problems requires a knowledge of nonequilibrium statistical mechanics for spacetime and quantum field dynamics. We also believe that *nonlocality* plays an important role in the entropy and information problem. It was for these reasons that we embarked on developing a formalism for treating nonequilibrium fields [1] in curved spacetime, searching for a way to define the entropy of quantum fields [61, 62], as well as seeking a physical understanding of Hawking radiation in terms of scale transformations [63, 64, 65]. The information flow in an interacting quantum field or between spacetime and matter fields cannot be fully captured by local observations. The correlation hierarchy scheme can incorporate and systemize the information content of the field and its dynamics more accurately. We are now applying it to some simple particle-field models to understand the physics [66], with the aim of treating the corresponding problems for black holes.

At a deeper theoretical level, our program of research has been to add a statistical mechanical dimension for quantum field theory. Approaching a problem treated ordinarily with quantum field theoretical emphasis with statistical mechanics viewpoints can bring in refreshingly new and valuable physical insights. Likewise, the many powerful conceptual and computational tools of field theory can enrich greatly both the theoretical and applied problems of equilibrium and nonequilibrium statistical physics. This effort has proven to

be extremely fruitful in many areas of physics. Quantum gravity, early universe and black hole physics are arenas where the laws of physics are pushed to the limit. Taking up such a challenge for these problems will be even more stimulating and rewarding.

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